哈尔滨工业大学航天学院力学系

高等弹性动力学第5讲

Biot孔隙介质的几个弹性系数

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石油勘探经常需要孔隙介质岩石模型

石油天然气存于孔隙岩石。 孔、渗、饱是描述储层的基本参数。

孔隙介质中弹性波的一种传播模型——**Biot** 波动理论

著名力学家，铁木辛柯奖得主

Maurice A. Biot 1905-1985

建立了含液体孔隙岩石的静力学模型（**1941**）

建立了井孔声波的力学模型，最早计算了井孔声场的伪瑞利波、斯通利波的频散曲线（**1952**）

Biot's equations for waves in porous media (1956-1962)
建立了描述孔隙介质流体-固体耦合运动的波动力学，预测了慢纵波的存在.

Biot Conferences on Poromechanics

——致力于研究孔隙介质力学与声学的国际会议

5th Biot Conference, Vienna, 2013，胡恒山组3篇论文

4th 2009 Columbia U., USA

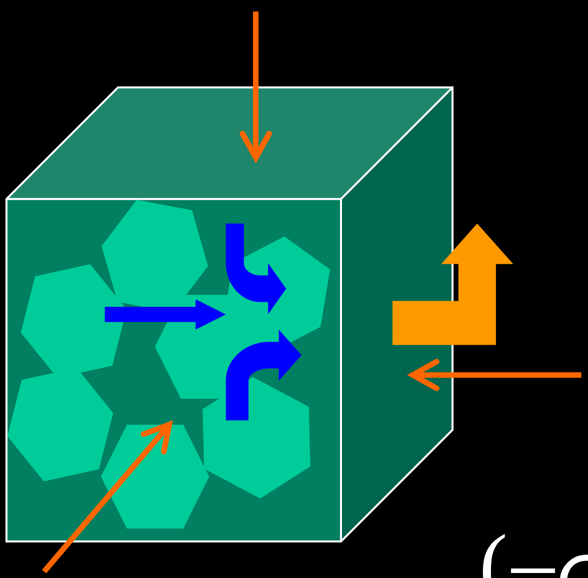
3rd 2005 Oklahoma, USA， 胡恒山宣读论文

$$\varepsilon = a_{11}\sigma + a_{12}p$$
$$\varsigma = a_{21}\sigma + a_{22}p$$

$$\varepsilon = \frac{1}{K}\sigma + a_{12}p$$
$$\varsigma = a_{21}\sigma + a_{22}p$$

$$\delta\sigma \neq 0$$

$$a_{11} = \frac{1}{K} = \left.\frac{\delta\varepsilon}{\delta\sigma}\right|_p$$



$$p = 0$$

$$(-\sigma) \Rightarrow (-\varepsilon)\uparrow$$

排水体积压缩系数

围压增大时，样品体积减小

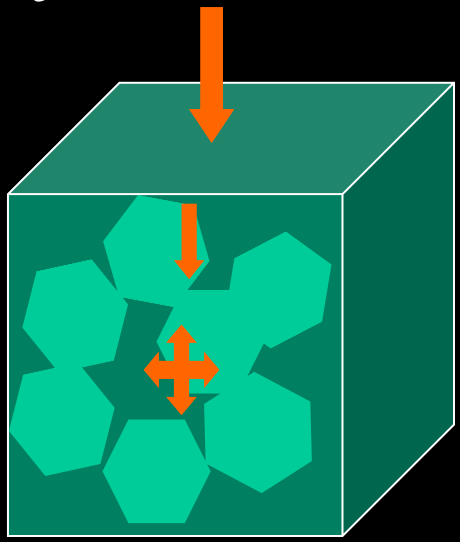
$$\frac{1}{K}$$

Drained Compression Coefficient $\frac{1}{K_d}$

$$\varepsilon = a_{11}\sigma + a_{12} p$$
$$\varsigma = a_{21}\sigma + a_{22} p$$

$$\varepsilon = \frac{1}{K}\sigma + \frac{1}{H^W} p$$
$$\varsigma = a_{21}\sigma + a_{22} p$$

$\varsigma \neq 0$



$$a_{12} = \frac{1}{H^W} = \left.\frac{\delta\varepsilon}{\delta p}\right|_{\sigma}$$

$$\sigma = 0$$

$$p\uparrow \;\Rightarrow\; \varepsilon\uparrow$$

围压不变，
增加孔压 (注入流体), 则样品膨胀
孔隙弹性膨胀系数

$$\frac{1}{H^W}$$

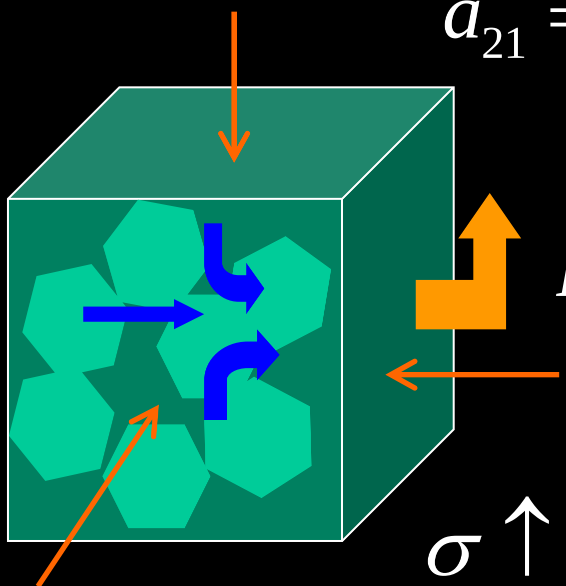
poroelastic expansion coefficient

like Heat expansion

$$\varepsilon = a_{11}\sigma + a_{12}p$$
$$\varsigma = a_{21}\sigma + a_{22}p$$

$$\varepsilon = \frac{1}{K}\sigma + \frac{1}{H^W}p$$
$$\varsigma = \frac{1}{H_1^W}\sigma + a_{22}p$$

$\delta\sigma \neq 0$



$$a_{21} = \frac{1}{H_1^W} = \frac{\delta\varsigma}{\delta\sigma}\bigg|_p$$

$p = 0$

$\sigma \uparrow \;\rightarrow\; \varsigma \uparrow$

围压增加，$(-\sigma)\uparrow \;\rightarrow\; (-\varsigma)\uparrow$
流体流出

$$\frac{1}{H_1^W} = \frac{1}{H^W}$$

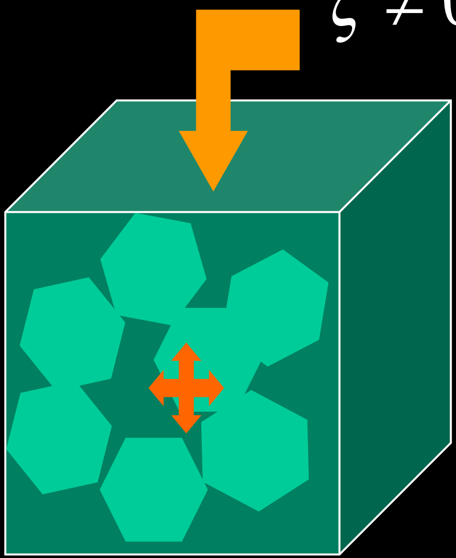
围压作用下，开孔样品流体含量降低系数

因骨架自身可压缩，
表观体积变化大于挤出的流体体积

$$\varepsilon = a_{11}\sigma + a_{12}p \quad \Rightarrow \quad \varepsilon = \frac{1}{K}\sigma + \frac{1}{H^{w}}p$$

$$\varsigma = a_{21}\sigma + a_{22}p \quad \Rightarrow \quad \varsigma = \frac{1}{H_1^{w}}\sigma + \frac{1}{R}p$$

$\varsigma \neq 0$



$\sigma = 0$

$p \uparrow \;\Rightarrow\; \varepsilon \uparrow$

围压不变，

增加孔压与注入流体同步

$$\frac{1}{R} = \left.\frac{\delta \varsigma}{\delta p}\right|_{\sigma} = a_{22}$$

unconstrained
specific
storage
coefficient

无约束的比储系数

$$\varepsilon = a_{11}\sigma + a_{12} p$$
$$\varsigma = a_{21}\sigma + a_{22} p$$

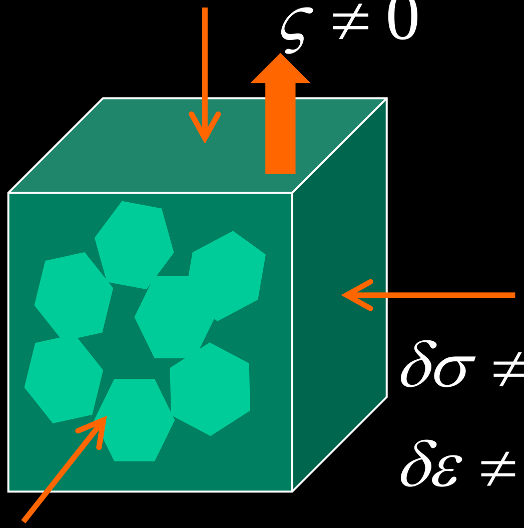
$$\varepsilon = \frac{1}{K}\sigma + \frac{1}{H^W} p$$
$$\varsigma = \frac{1}{H_1^W}\sigma + \frac{1}{R} p$$

$$\varepsilon = \frac{1}{K}\sigma + \frac{\alpha}{K} p$$
$$\varsigma = \frac{\alpha}{K}\sigma + \frac{\alpha}{KB} p$$

(2.16-18)
H.F.Wang

$p = 0$

$\varsigma \neq 0$



$\delta\sigma \neq 0$

$\delta\varepsilon \neq 0$

$$\frac{1}{K} = \left.\frac{\delta\varepsilon}{\delta\sigma}\right|_p$$

$$\frac{1}{H^W} = \left.\frac{\delta\varsigma}{\delta\sigma}\right|_p$$

$$\frac{K}{H^W} = \left.\frac{\delta\varsigma}{\delta\varepsilon}\right|_p$$

$$\left.\frac{\varsigma}{\varepsilon}\right|_p = \frac{C}{M} = \alpha$$

$$\frac{K}{H^W} = \frac{C}{M} = \alpha$$

（未封膜，开孔）孔压不变。

增大围压可使体积减小、流体流出。

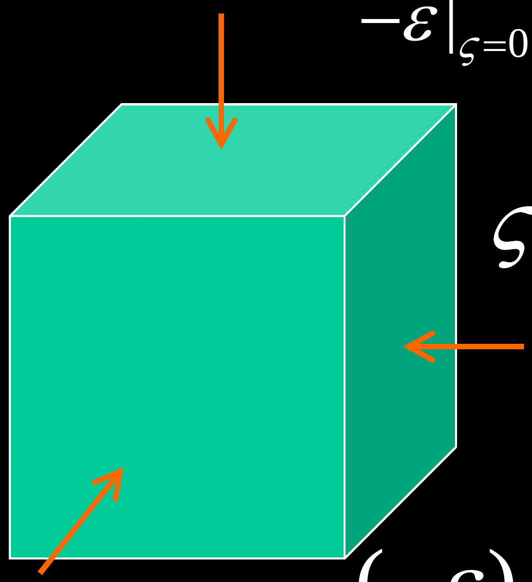
α 是流体含量增量与体膨胀量之比

围压作用下，开孔样品排出的流体体积小于样品减少的表观体积

$$p = -\varepsilon C + M\varsigma$$

$$\varepsilon = \nabla \cdot \mathbf{u}\ ,\quad \varsigma = -\nabla \cdot \mathbf{w} \qquad\qquad \sigma = K_c\varepsilon - C\varsigma$$

$$C = \left.\frac{p}{-\varepsilon}\right|_{\varsigma=0} = -\left.\frac{\delta p}{\delta\varepsilon}\right|_{\varsigma}$$



$\varsigma = 0$

undrained compression

$(-\varepsilon)\uparrow \Rightarrow \boldsymbol{p}\uparrow$

C：包膜（不排水）条件下压缩岩样引起的孔隙流体压强增量

$$B = -\left.\frac{\delta p}{\delta\sigma}\right|_{\varsigma}$$

B 不排水条件下, 施加围压而导致的孔隙流体压强增量.

反映孔隙流体与固体骨架在承担围压中的贡献比例

$$B = -\left.\frac{\delta p}{\delta\varepsilon}\right|_{\varsigma} \times \left.\frac{\delta\varepsilon}{\delta\sigma}\right|_{\varsigma} = \frac{C}{K_c} = \frac{\alpha M}{K_c}$$

系数之间的关系

Skempton's [1954] coefficient B and the Biot andWillis [1957] constant α

$$0 < \alpha < \frac{1}{B}$$

$$\alpha B = \left(1 - \frac{K_d}{K_c}\right)$$

$$B = -\left.\frac{\delta p}{\delta \varepsilon}\right|_{\varsigma} \times \left.\frac{\delta \varepsilon}{\delta \sigma}\right|_{\varsigma} = \frac{C}{K_c} = \frac{\alpha M}{K_c}$$

$$K_C = \left.\frac{\delta \sigma}{\delta \varepsilon}\right|_{\varsigma} = \alpha C + K_b$$

慢纵波对应的渗流位移与固相位移之比

$$\beta = -\frac{1}{B}\left(1 + \frac{4}{3}\frac{G}{K_c}\right)$$

Darcy's Law

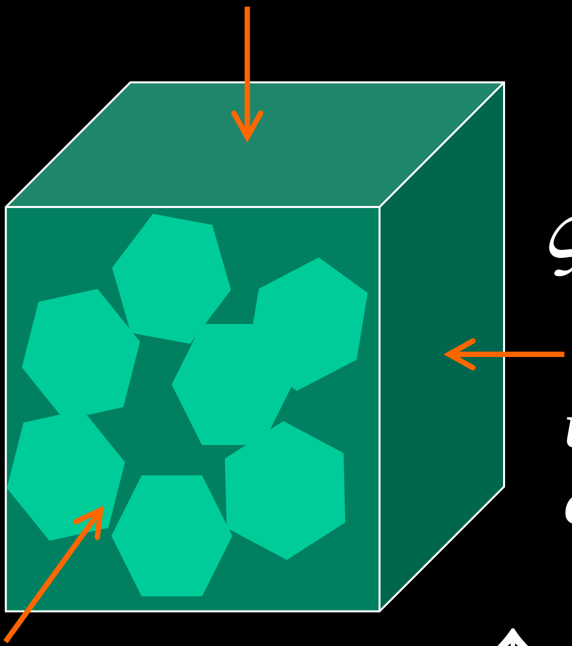
$$\dot{\mathbf{w}} = \frac{\kappa}{\eta}\left(\left(-\nabla p\right) + Inertial\ \ force\ on\ \ pore\ \ fluid\right)$$

$$-\rho_f \ddot{\mathbf{u}}$$

$$\dot{\mathbf{w}} = \frac{\kappa}{\eta}(-\nabla p + i\omega\rho_f \dot{\mathbf{u}})$$

Biot's equations

$$\varepsilon = \nabla \cdot \mathbf{u}\,, \quad \varsigma = -\nabla \cdot \mathbf{w} \qquad \sigma = K_c \varepsilon - C\varsigma$$



$$\varsigma = 0$$

undrained compression

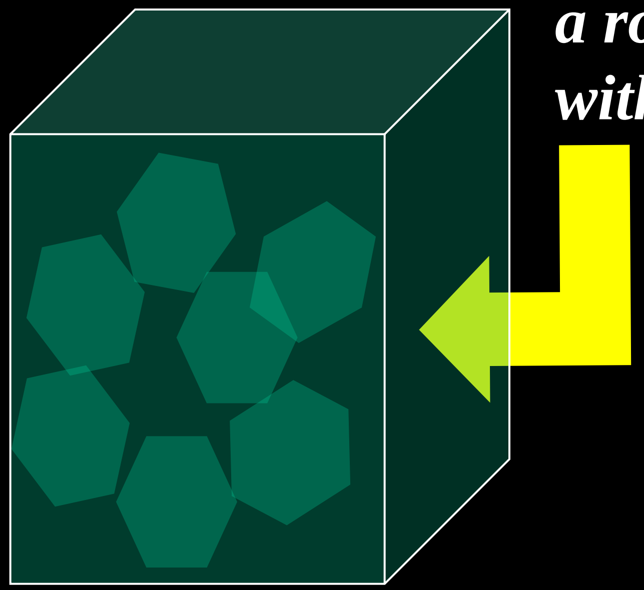
$$(-\varepsilon)\uparrow \;\Rightarrow\; \boldsymbol{p}\uparrow$$

包膜（不排水）条件下，
压缩岩样而增加的孔隙流体压强

$$p = -\varepsilon C + M\varsigma$$

$$C = \left.\frac{p}{-\varepsilon}\right|_{\varsigma=0} = -\left.\frac{\delta p}{\delta \varepsilon}\right|_{\varsigma}$$

$$K_C = \left.\frac{\delta \sigma}{\delta \varepsilon}\right|_{\varsigma} = \alpha C + K_b$$



$$\varepsilon = 0$$

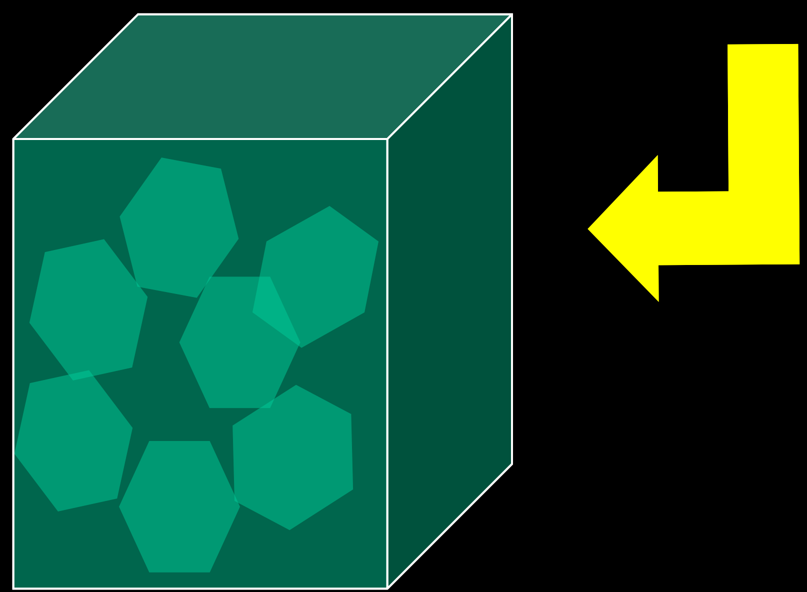
$$\varsigma \uparrow \;\Rightarrow\; p \uparrow$$

M： 保持岩样表观体积不变，
因注入流体而增大的流体压强

$$p = -\varepsilon C + M\varsigma$$

$$M = \left.\frac{p}{\varsigma}\right|_{\varepsilon=0} = \left.\frac{\delta p}{\delta \varsigma}\right|_{\varepsilon}$$

input fluid into a rock sample within a rigid shell



$$\varepsilon = 0$$

$$\varsigma \uparrow \;\Rightarrow\; \boldsymbol{p} \uparrow$$

M 保持岩样表观体积不变，因注入流体而增大的流体压强

$$p = -\varepsilon C + M\varsigma$$

$$S_\varepsilon = \frac{1}{M} = \left.\frac{\delta\varsigma}{\delta p}\right|_\varepsilon$$

$$S_\sigma = \frac{1}{R} = \left.\frac{\delta\varsigma}{\delta p}\right|_\sigma$$

与具有自由边界样品相比，具有刚性边界的样品（对注入同样多的流体）呈现出更大的抵抗力。

$$S_\varepsilon < S_\sigma \;\Rightarrow\; M > R$$

$$\frac{1}{M}=\frac{1}{R}-\frac{K}{\left(H^{W}\right)^{2}}$$

$$S_{\varepsilon}=S_{\sigma}-\frac{K}{\left(H^{W}\right)^{2}}$$

$$p=-\varepsilon C+M\varsigma$$

$$S_{\varepsilon}=\frac{1}{M}=\left.\frac{\delta\varsigma}{\delta p}\right|_{\varepsilon}$$

$$S_{\sigma}=\frac{1}{R}=\left.\frac{\delta\varsigma}{\delta p}\right|_{\sigma}$$

注入同样多的流体,刚性边界会导致更大的流体压强增量。

$S_{\varepsilon}<S_{\sigma}$ ⇒ $M>R$

Biot's equations

$$\varepsilon = \nabla \cdot \mathbf{u} \,, \quad \varsigma = -\nabla \cdot \mathbf{w}$$

$$\sigma = K_c \varepsilon - C\varsigma$$

$$p = -\varepsilon C + M\varsigma$$

$$p = 0$$

$$\varepsilon C = M\varsigma$$

$$\left.\frac{\varsigma}{\varepsilon}\right|_p = \frac{C}{M} = \alpha \le 1$$

$$C = \left.\frac{p}{-\varepsilon}\right|_{\varsigma=0} = -\left.\frac{\delta p}{\delta \varepsilon}\right|_{\varsigma}$$

$$C = -\left.\frac{\delta P}{\delta \varepsilon}\right|_{\varsigma} \left.\frac{\delta p}{\delta P}\right|_{\varsigma} = BK_c$$

$$M = \left.\frac{p}{\varsigma}\right|_{\varepsilon=0} = \left.\frac{\delta p}{\delta \varsigma}\right|_{\varepsilon}$$

$$M = C/\alpha \ge C$$

$$\frac{1}{K} = \left.\frac{\delta\varepsilon}{\delta\sigma}\right|_p \qquad \frac{1}{H^W} = \left.\frac{\delta\varsigma}{\delta\sigma}\right|_p$$

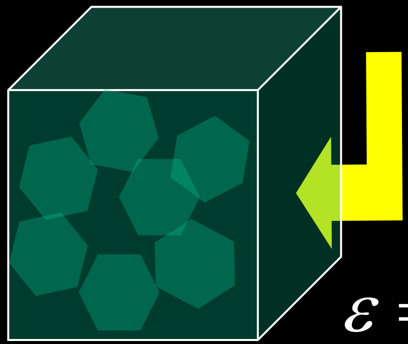
$$\frac{K}{H^W} = \left.\frac{\delta\varsigma}{\delta\varepsilon}\right|_p$$

$$p = -\varepsilon C + M\varsigma$$

$$\left.\frac{\varsigma}{\varepsilon}\right|_p = \frac{C}{M} = \alpha$$

$$\frac{K}{H^W} = \frac{C}{M} = \alpha$$

$\varsigma \neq 0$



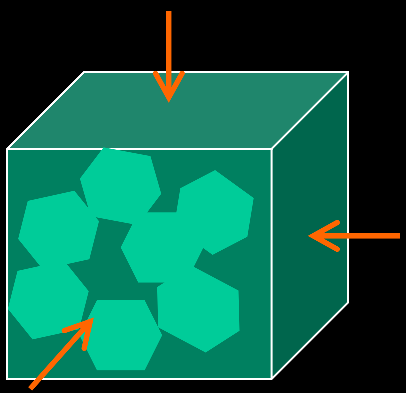
$\varepsilon = 0$

input fluid into a rock sample within a rigid shell

$$M = \left.\frac{p}{\varsigma}\right|_{\varepsilon=0} = \left.\frac{\delta p}{\delta \varsigma}\right|_{\varepsilon}$$

$$M = \frac{1}{\dfrac{\phi}{K_f} + \dfrac{\alpha - \phi}{K_s}} = \frac{K_f K_s}{\phi K_s + (\alpha - \phi) K_f}$$

$$K_C = \left.\frac{\delta \sigma}{\delta \varepsilon}\right|_{\varsigma} = \alpha C + K_b$$



$\varsigma = 0$

undrained compression

$$B = -\left.\frac{\delta p}{\delta \sigma}\right|_{\varsigma} = \left.\frac{\Delta p}{\Delta P}\right|_{\varsigma=0} = \frac{\alpha M}{K_c} = \frac{\alpha M}{\alpha^2 M + K_b}$$

$$B = \frac{\alpha}{\dfrac{\phi K_c}{K_f} + \dfrac{(\alpha - \phi) K_c}{K_s}} = \frac{\alpha K_f K_s}{\phi K_s K_c + (\alpha - \phi) K_f K_c}$$

Skempton系数*B*

Wang, 2000

$$B = \left.\frac{\Delta p}{\Delta P}\right|_{\xi=0} = \frac{\alpha M}{K_c} \qquad B = \left.\frac{\Delta p}{\Delta P}\right|_{\xi=0} = \frac{\alpha M}{K_c}, \quad C = K_c B$$

$$\alpha = 1 - \frac{K_b}{K_s}, \qquad M = \frac{K_f K_s}{\phi K_s + (\alpha - \phi) K_f},$$

$$K_c = K_b + \alpha^2 M$$

$$K_s, K_f, \phi \implies K_s, K_f, \phi, K_b \implies \alpha, M, K_b \implies K_c$$

$$\mu \implies \rho V_S^2 \qquad \frac{4}{3}\mu + K_c = H \implies \rho V_P^2$$

表观体应变的有效应力定理

一块孔隙介质，在两种不同的围压与孔隙组合下分别承受不排水压缩，那么，表观体积改变量只取决于有效应力。

$$P_{ij} = P_{ij} - \alpha p \delta_{ij}$$

特别的，只要有效应力相等，表观体积改变量都等于围压不为零而孔压为零的组合（即排水条件）下的体积改变量。

波动方程向扩散方程的退化

$$\nabla^2\mathbf{E} = \mu\sigma\frac{\partial\mathbf{E}}{\partial t} + \mu\varepsilon\frac{\partial^2\mathbf{E}}{\partial t^2}$$

$\varepsilon\omega \ll \sigma$ 良导体

$$\nabla^2\mathbf{E} = \mu\sigma\frac{\partial\mathbf{E}}{\partial t}$$

$$k = \omega\sqrt{\mu\varepsilon'}, \quad c = \frac{1}{\sqrt{\mu\varepsilon'}}$$

$$\varepsilon' = \varepsilon + i\frac{\sigma}{\omega}$$

$$\nabla^2\mathbf{E} + k^2\mathbf{E} = 0$$

$$k^2 \approx i\omega\mu\sigma = \frac{i\omega}{C_D}, \quad k = \frac{1+i}{\sqrt{2}}\sqrt{\omega\mu\sigma}, \quad Q^{-1} = 2\frac{\mathbf{Im}(k)}{\mathbf{Re}(k)} = 2$$

$$c^2 \approx \frac{-i\omega}{\mu\sigma} = -i\omega C_D$$ 此速度无传播意义

$$\nabla^2 T - \frac{1}{C_D}\frac{\partial T}{\partial t} = 0, \quad C_D = \frac{k}{\rho c}, \quad q = -k\nabla T$$

热流密度与温度梯度呈正比

扩散率=热传导系数与（密度与比热的积）之比

孔隙介质慢纵波

$$V_{P2}^2 \approx -i\omega C_D$$

$$C_D = \frac{\kappa K_f}{\eta\phi}\left(1 + \frac{K_f}{\phi\left(K_b + \frac{4}{3}N\right)}\left\{1 + \frac{1}{K_s}\left[\frac{4}{3}N\left(1 - \frac{K_b}{K_s}\right) - K_b - \phi\left(K_b + \frac{4}{3}N\right)\right]\right\}\right)^{-1}$$

Chandler and Johnson,1981

谢 谢 ！

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