Adaptive backstepping fault-tolerant control for flexible spacecraft with unknown bounded disturbances and actuator failures

Ye Jiang, Qinglei Hu*, Guangfu Ma
Department of Control Science and Engineering, Harbin Institute of Technology, Harbin, 150001, China

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A B S T R A C T

In this paper, a robust adaptive fault-tolerant control approach to attitude tracking of flexible spacecraft is proposed for use in situations where there are reaction wheel/actuator failures, persistent bounded disturbances and unknown inertia parameter uncertainties. The controller is designed based on an adaptive backstepping sliding mode control scheme, and a sufficient condition under which this control law can render the system semi-globally input-to-state stable is also provided such that the closed-loop system is robust with respect to any disturbance within a quantifiable restriction on the amplitude, as well as the set of initial conditions, if the control gains are designed appropriately. Moreover, in the design, the control law does not need a fault detection and isolation mechanism even if the failure time instants, patterns and values on actuator failures are also unknown for the designers, as motivated from a practical spacecraft control application. In addition to detailed derivations of the new controller design and a rigorous sketch of all the associated stability and attitude error convergence proofs, illustrative simulation results of an application to flexible spacecraft show that high precise attitude control and vibration suppression are successfully achieved using various scenarios of controlling effective failures.

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1. Introduction

Accurate and reliable control is one of the most important problems in spacecraft design. Although the missions of space vehicles and their attitude requirements vary greatly, high pointing accuracy and fault tolerance are important parts of the overall design problem for the spacecraft control system. However, the orbiting attitude slewing or tracking operation will introduce certain levels of vibration to flexible appendages, which will deteriorate its pointing performance. The dynamics of spacecraft are time varying and highly nonlinear, and they are affected by various disturbances coming from the environment and by insufficient knowledge of the system parameters such as the inertia matrix, which are usually not well known. Moreover, actuators may fail during system operation, and the actuator failures are often uncertain in the sense that it is not known when, by how much, and how many actuators fail. All these aspects in a realistic environment create considerable difficulty in the design of an attitude control system for adequate performance and stability, especially when all these issues are treated simultaneously.

In the face of various environmental disturbances and the increasingly complex and highly uncertain nature of spacecraft dynamical systems requiring controls, many studies related to attitude control of flexible spacecraft have been done, and robust linear and nonlinear control systems have been designed. Control laws based on linearization and nonlinear inversion have been presented in Ref. [1]. Optimal and nonlinear control systems for the control of flexible spacecraft have been developed in Refs. [2, 3]. Based on Lyapunov stability and dissipativity theory, dynamics attitude control laws for flexible spacecraft have been designed in Refs. [4, 5]. Variable structure control (VSC) for certain types of disturbances and uncertainties is also attractive for spacecraft control problems, and many related works have been attempted in Refs. [6–9] and the references therein. However, these design methods require information on the bounds on the uncertainties/disturbances for the computation of the control gains. Unlike these methods, nonlinear adaptive control methods do not require these bounds; instead, they include an adaptation mechanism for tuning the time-varying controller gains. A variety of adaptive spacecraft controllers have been developed [10–12]. Research has also been focused on the combination of VSC and adaptive control to develop simple and adaptive robust spacecraft controllers that work for a wide range of practical systems [13–16]. Recently, the design of a composite adaptive control system using the backstepping technique has been also considered [17]. The advantage of adaptive backstepping compared with other control methods lies in its design flexibility, due to its recursive use of Lyapunov functions. The control torque is designed for each integrator

* Corresponding address: P.O. Box 327, Department of Control Science and Engineering, Harbin Institute of Technology, 92 West Da-Zhi Street, Harbin, Heilongjiang Province, 150001, China. Tel.: +86 451 86413411x8606; fax: +86 451 86418378.
E-mail address: huqinglei@hit.edu.cn (Q. Hu).
level in the system, and this enables the possibility to compensate for destabilizing nonlinearities, while stabilizing nonlinearities can be exploited. Moreover, this approach essentially combines the adaptive schemes for faster convergence of the estimated parameters and the tracking error. Several authors have considered the design of an adaptive supervisory control system using the backstepping method [18,19]. Using backstepping controllers with or without an adaptive mechanism has also been developed for attitude stabilization and tracking of rigid spacecraft in Refs. [20–22] and the references therein. However, most (if not all) of the previous research work can hardly be extended to a flexible spacecraft system when the effect of the motion of the elastic appendages is taken into account, especially without a priori knowledge of the bound of the modal variables.

Another important problem encountered in practice for spacecraft control system design is that of control failures. As is well known, control failures can caused control system performance deterioration and lead to instability and even catastrophic accidents. Therefore, it is essential to maintain high reliability for spacecraft control system design against possible control faults. A fault-tolerant controller could be desirable to find a solution to meet the desired objective in addition to delivering the desired moments. Fault tolerant control (FTC) is an area of research that emerges to increase availability by specifically designing control algorithms capable of maintaining stability and performance despite the occurrence of faults, and it has received considerable attention from the control research community and aeronautical engineering in the past couple of decades. In particular, as introduced in Ref. [23], Boškovic et al. used a multiple models method to detect and isolate actuator faults for a spacecraft attitude control system. Based on a dynamically driven recurrent neural network architecture, an FDI strategy was proposed for the satellite's attitude control system when thruster failures occurred [24]. Chen and Saif [25] presented a fault diagnosis approach in a satellite system for identifying thruster faults by using an iterative learning observer. In [26], a robust FDI method based on neural state space models was applied to a satellite attitude control subsystem, and the robustness, sensitivity and stability properties of this method were investigated. In Ref. [27], the authors use the dynamics inversion and time-delay theory to design a passive fault-tolerant controller for a rigid satellite with four reaction wheels to achieve attitude tracking control. To take into account the redundant thrusters, an indirect adaptive fault-tolerant control for attitude tracking of rigid spacecraft is proposed in the presence of unknown uncertainties, disturbances and actuator failures, in which a bounded parameter of the lumped perturbations is introduced to be updated on-line [28].

In this paper, an attempt is made to provide an adaptive FTC strategy for a flexible spacecraft with redundant actuators, such as four reaction wheels which are commonly used for attitude control, which addresses the aforementioned issues. The proposed control strategy is based on backstepping sliding mode control theory and it is applied to a flexible spacecraft suffering from unknown faults of the reaction wheels, external disturbances, and unknown inertia matrix of the spacecraft. A key feature of the proposed strategy is that the design of the FTC is done independently of the information about the faults, and the measurements of the flexibility variables are not needed as well. A sufficient condition is also provided such that the proposed fault-tolerant controller guarantees that the closed loop is semi-global input-to-state stable by the Lyapunov-like stability analysis. The effects due to the flexible elements of the spacecraft could also be treated provided the robustness conditions on the control gains guarantee the satisfaction, exactly or approximately, of the control requirements, under appropriate conditions on the partial system parameters. Finally, applications are carried out on an orbiting spacecraft with flexible appendages.

The paper is organized as follows. The next section states the flexible spacecraft modeling and control problems. Using the input-to-state stability characteristic, the adaptive backstepping sliding mode fault-tolerant control law is derived in Section 4. Next, the results of numerical simulations demonstrate various features of the proposed control law. Finally, the paper is completed with some concluding comments.

2. Mathematical model of flexible spacecraft

2.1. Kinematic equation

The unit quaternion is adopted to describe the attitude of the spacecraft for global representation without singularities [29]. The unit quaternion $\bar{q}$ is defined by

$$\bar{q} = \begin{bmatrix} \cos(\Phi/2) \\ n \sin(\Phi/2) \end{bmatrix} = \begin{bmatrix} q_0 \\ q \end{bmatrix}$$

(1)

where $n$ is the Euler axis, $\Phi$ is the Euler angle, and $q_0$ and $q$ are the scalar and vector components of the unit quaternion, respectively; they are subject to the constraint $q^Tq + q_0^2 = 1$. Then the kinematic equation in terms of the unit quaternion can be given by

$$\dot{\bar{q}}_0 = 1/2 \left[ q_0 I + S(q) \right] \omega$$

(2)

where $\omega \in R^3$ is the angular velocity of a body-fixed reference frame of a spacecraft with respect to an inertial reference frame expressed in the body-fixed reference frame, $I \in R^{3 \times 3}$ represents the identity matrix, and $S(q)$ denotes a skew-symmetric matrix which is given by

$$S(q) = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix}$$

(3)

2.2. Relative attitude error kinematics

Let $\bar{q}_e = [q_{0e} \ q^T]_T$ denote the relative attitude error from a desired reference frame to the body-fixed reference frame of the spacecraft; then one may have

$$\bar{q}_e = \bar{q} \otimes \bar{q}_d^{-1} = [q_{0e} \ q^T]_T$$

(4)

where $\bar{q}_d^{-1}$ denotes the inverse of the desired quaternion $\bar{q}_d$ with the definition $\bar{q}_d^{-1} = [q_{0d} - q^T]_T$ and $\otimes$ is the operator for quaternion multiplication, which is defined by

$$\bar{q}_a \otimes \bar{q}_b = \begin{bmatrix} q_{0a}q_{0b} - q^T q_b \\ q_{0a}q_b + q_{0b}q_a - S(q_a)q_b \end{bmatrix}$$

(5)

for any given two groups of quaternions of $q_a$ and $q_b$. As a result, the relative attitude error can be obtained by

$$\begin{bmatrix} \dot{q}_{0e} \\ \dot{q}_e \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -q^T \\ q_{0e}I + S(q_e) \end{bmatrix} \omega(t) - R_d \omega_d(t)$$

(6a)

or

$$\begin{bmatrix} \dot{q}_{0e} \\ \dot{q}_e \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -q^T \\ q_{0e}I + S(q_e) \end{bmatrix} \omega(t) \text{ for } \omega_d = 0$$

(6b)

where $R_d$ is the rotation matrix from the desired reference frame to the body-fixed reference frame, and $\omega_d$ is the angular velocity of the desired reference frame with respect to the inertial reference frame expressed in the desired reference frame. Note that in this paper we consider the case $\omega_d = 0$, for convenience, to develop the control law. It is worth noting that the developed approach can be generalized to the case $\omega_d \neq 0$ by properly changing the designed controller form (or by adding the terms related to $\omega_d$ and $\omega_d$, respectively, as the feed-forward control inputs of the attitude control system).
2.3. Flexible spacecraft dynamics

Under the assumption of small elastic displacements, the dynamic equations of a spacecraft with flexible appendages can be found in Ref. [9] and the references therein, and are given by

\begin{align}
J\ddot{\omega} + \delta \dot{\eta} & = -\omega \times (J\omega + \delta \dot{\eta}) + u(t) + d(t) \quad (7a) \\
\ddot{i} + C_i \dot{i} + K_i \ddot{i} + \delta \ddot{\omega} & = 0 \quad (7b)
\end{align}

where \( J \) is the symmetric inertia matrix of the whole structure, \( \delta \) is the coupling matrix between the elastic and rigid structure, \( \eta \) is the modal coordinate vector, \( u(t) \) is control torque acting on the main body and generated by, for example, the reaction wheels, and \( d(t) \) is the external disturbance including environmental torques such as gravitational torque, torque as a result of solar radiation, and magnetic effects, etc.; the matrices \( C = \text{diag}(2\zeta_i A_i^2, i = 1, 2, \ldots, N) \) and \( K = \text{diag}(A_i, i = 1, 2, \ldots, N) \) are the damping and stiffness matrices, respectively, in which \( N \) is the number of elastic modes considered, \( A_i^2 \) is the natural frequency, and \( \zeta_i \) is the corresponding damping ratio.

Remark 1. The above dynamics of the spacecraft are obtained by computing the kinetic and potential energies and then applying the Lagrange equations with the assumption of small elastic displacement approximation. This simplified equation is easy to manipulate and more suitable for control law design. Of course, the exact model, which is time varying and more difficult to handle, can be used instead for verifying the effectiveness of the control law derived on the basis of the simplified model, to accomplish the rational maneuver and vibration reduction for the closed-loop simulation later.

Let us consider that an actuator fault occurs. In particular, consider the situation in which the actuator loses complete or partial control power; a bias fault is also considered [27]. Then the general nonlinear spacecraft attitude dynamics model in Eq. (7a) with four reaction wheels can be rewritten as

\begin{align}
(\mathbf{I} - D J w D^T)\ddot{\omega} + \delta \dot{\eta} & = -\omega \times (\mathbf{I} \omega + D J w \Omega + \delta \dot{\eta}) \\
+ D E u + D f + d \quad (8a) \\
J w \ddot{\Omega} & = -u(t) - J w D^T \dot{\omega} \quad (8b) \\
\ddot{i} + C_i \dot{i} + K_i \ddot{i} + \delta \ddot{\omega} & = 0 \quad (8c)
\end{align}

where \( J_w \) is the inertia matrix of the reaction wheels, \( \Omega \) is the wheel angular velocity, \( D \in \mathbb{R}^{3 \times 4} \) is the reaction wheel orientation matrix (for a given spacecraft, \( D \) is available and can be made full row by placing the wheel at a certain location and direction on the spacecraft, i.e., \( \text{rank}(D) = 3 \)), vector \( f \) is the modeling additive faults (e.g., bias fault) and the diagonal \( E \) is the multiplicative faults (e.g., reduced control torque) of the reaction wheels. Here \( E \) is defined as

\[
E = \text{diag} [e_1, e_2, e_3, e_4], \quad 0 \leq e_i \leq 1, \quad i = 1, 2, 3, 4.
\]

It is assumed that the angular velocity \( \Omega \) of the reaction wheels are within the saturation limit and the input control torque \( u(t) \) is unbounded.

Throughout the remainder of this paper, the following three assumptions are made:

Assumption 1. The inertia matrix \( J \) defined in Eq. (7a) is positive definite symmetric and bounded during the entire orbiting operation, but is unknown.

Assumption 2. The external disturbance \( d(t) \) in the spacecraft system (7a) is unknown but bounded.

Assumption 3. The control effectiveness matrix \( E \) and vector \( f \) are also assumed to be unknown during the entire orbiting operation, but \( \| D f(t) \| \) is bounded.

Remark 2. For Assumption 1, the structural parameters are supposed to be poorly known, and are constant or can vary during spacecraft operations. In both cases, since their variation is assumed to be slow with respect to the spacecraft dynamics, their derivatives are or can be considered zero. Even if bounds are assumed, they are not used in the control system design, which is a reasonable assumption in practice. For Assumptions 2 and 3, they are feasible from the practical point of view for a certain orbiting spacecraft. Note that the notation \( \| x \| \) in this paper denotes the Euclidean norm of vector \( x \), and \( \| X \| \) is the induced two-norm of a matrix \( X \).

3. Derivation of an adaptive backstepping fault-tolerant control law

In this work, the control objectives are to achieve high precision of attitude tracking and vibration reduction of a flexible spacecraft in the presence of external disturbances, parameter uncertainties and unknown actuator faults. For this, a robust adaptive backstepping control strategy is investigated in this paper. Adaptive backstepping is a recursive Lyapunov-based scheme and the idea of it is to design a controller recursively by considering some of the state variables as ‘virtual controls’ and designing intermediate control laws for them. The advantage of adaptive backstepping compared with other control methods lies in its design flexibility, due to its recursive use of Lyapunov functions such that cancellations of useful nonlinearities are avoided and often additional nonlinear terms are introduced to improve the transient performance.

To give a clear idea of such controller design procedure, the following variables are defined:

\[
x_1 = q_e, \quad x_2 = \omega - \varphi(q_e) \quad (10a)
\]

where \( \varphi(q_e) \) is the intermediate, or virtual, control law and will be defined below.

Step 1. By considering \( x_2 \) as the virtual control variable, the derivative of Eq. (10a) is given as

\[
\dot{x}_1 = \dot{q}_e = -\frac{1}{2} (q_{0e} f - S(q_e) \omega) \omega(t). \quad (11)
\]

Note that, in this step, the task is to stabilize Eq. (11) with respect to the Lyapunov function

\[
V_1 = \frac{1}{2} \left( x_1^T x_1 + (1 - q_{0e})^2 \right) = (1 - q_{0e}), \quad (12)
\]

and the time derivative of \( V_1 \) can be given by

\[
\dot{V}_1 = \frac{1}{2} x_1^T (x_2 + \varphi(\omega)) = \frac{1}{2} x_1^T (x_2 - k_1 x_1) = -\frac{1}{2} k_1 x_1^T x_1 + \frac{1}{2} x_1^T x_2 \quad (13)
\]

in which \( \varphi(\omega) = -k_1 x_1 \) is selected and \( k_1 > 1 \) is the designed parameter.

Step 2. In this step, an adaptive backstepping control law is designed using a sliding mode control scheme. Essential to the design of a sliding mode control law is the selection of a switching surface; then the control law is designed such that all the trajectories are attracted towards this surface, and after reaching the surface they slide on it. The structure of the controller changes when the trajectory crosses the switching surface. By taking the error quaternion

\[
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\]


\[ \gamma_1 \|z\|^2 \leq V_2 \leq \gamma_2 \|z\|^2 \] (23)
\[ + \| F \| \| \dot{\alpha} \| \| \sigma \| - \frac{1}{\mu_1} \dot{\lambda} (\mu_1 \| F \| \| \sigma \|) \]
\[ \leq -\frac{1}{2} k_1 x_1^T x_1 - \xi^T Q \xi - k_2 \sigma^T \sigma + \frac{1}{2} x_2^T x_2 \]
\[ + \left( \left\| \begin{bmatrix} -\delta & C \delta \end{bmatrix} \right\| + \left\| \begin{bmatrix} \delta^T K & \delta^T C \end{bmatrix} \right\| + (k_1 + \beta) \left\| \delta^T \right\| \right) \| \omega \| \| \xi \| + \| \sigma \| \| D_f + d \| \]
To this end, before we state our main result, let us first state the following definition of input-to-state stability. [30]

**Definition.** The closed-loop system (6b), (8) and (27) is semi-globally input-to-state stable (ISS) with respect to state $z(t)$ and input $(d(t) + Df)$ if for any positive constants $\Delta_2$ and $\Delta_{(d+Df)}$ there exist $k_1$, $k_2$ and $\lambda$ such that, given $\|z(0)\| < \Delta_2$ and $\sup_{0 \leq s \leq t} \|d(t) + Df(t)\| < \Delta_{(d+Df)}$, the following bound on the state $z(t)$ is guaranteed to hold:

$$
\|z(t)\| \leq \max \left\{ \chi \left( \|z(0)\|, t \right), \phi \left( \sup_{0 \leq r \leq t} \|d(r) + Df(r)\| \right) \right\},
$$

for some functions $\chi(\cdot) \in \mathcal{KL}$ and $\phi(\cdot) \in \mathcal{K}_\infty$, which are described in the following Remark.

**Remark 3.** A continuous function $\phi : [0, a) \to [0, \infty)$ belongs to class $\mathcal{K}$ if it is strictly increasing and $\phi(0) = 0$. It belongs to $\mathcal{KL}$ if $a = \infty$ and $\phi(r) \to \infty$ as $r \to \infty$. The function $\chi : [0, a) \times [0, \infty) \to [0, \infty)$ belongs to class $\mathcal{K}_\infty$ if for each fixed $s$ the mapping $\chi(r, s)$ belongs to class $\mathcal{K}$ with respect to $r$ and for each fixed $r$ the mapping $\chi(r, s)$ is decreasing with respect to $s$ and $\chi(r, s) \to 0$ as $s \to \infty$. See more details in Ref. [30].

Then, using similar arguments as in Ref. [30], we can show that whenever the bounds Eqs. (23) and (30) hold in the restricted region for $V_2$ and $V_3$, they can be transformed into the following bound on the system states:

$$
\|z\| \leq \max \left\{ 2 \frac{\sqrt{2}}{\gamma_1} e^{-\frac{\rho k M}{2}} \|z\| (0), \right\}
$$

(33)

And then we have the following statements.
Consider the flexible spacecraft system in Eqs. (27) and (8).

**Theorem 1.** Consider a spacecraft system involving actuator faults governed by Eq. (9) under Assumptions 1–3. Given any initial condition $z(0)$, there exist large enough control gains $k_1$, $k_2$, $\lambda$, and $\mu_1$ such that the closed-loop system (6b), (8) and (27) is semi-globally ISS from an external disturbance input $(d(t) + Df(t))$ to the state $z(t)$ with the bound restrictions $\Delta_x$ and $\Delta_d(t)$ on the state $z(t)$ and disturbance $(d(t) + Df(t))$, respectively, even in the presence of unknown actuator faults.

**Remark 4.** From the above analysis, because the matrix $E$ is not used in the control scheme, there is no need to include a health monitoring unit to identify or estimate which actuator is unhealthy. Knowledge of the degree of failure for each actuator is not even needed. The wheel fault accommodation/compensation is done automatically and adaptively by the proposed control algorithm. This feature is necessary to build affordable and effective fault-tolerant spacecraft control schemes.

**Remark 5.** From Eq. (27), the above stability proof is ensured as long as $DED^T$ is positive definite. That is to say that it is required that the number of active controls after failure should be greater than or at least equal to 3, i.e. the remaining active wheels are able to produce an efficient actuating torque vector for the spacecraft to perform the given maneuvers.

**Remark 6.** To prevent parameter drift and avoid singularities in the adaptive control law, several methods of robustifying the update laws have been suggested in the literature [31] over the years; the following modified update law is used in the later simulation study:

$$\dot{\hat{\alpha}} = -\mu_1 \mu_2 \hat{\alpha} + \mu_1 \frac{||F||}{\sigma} ||\sigma||$$

where $\mu_2$ is designed positive constant.

**Remark 7.** In practice, the sign function in the control law (27a) can be replaced by a saturation-like function to reduce the chattering as

$$u(t) = \frac{D^T}{\lambda} \left[ -k_2 \sigma - \left( ||S(\omega)DJ_w(\Omega + D^T\omega)|| + \frac{||\delta^T C \delta \omega|| + ||F|| \hat{\alpha}}{\sigma} \right) \right]$$

by introducing a small boundary $\epsilon > 0$.

In the above analysis, we assume that the disturbance $(d(t) + Df(t))$ is simply bounded, i.e., $(d(t) + Df(t)) \in L_2[0, \infty)$. We further suppose that $(d(t) + Df(t)) \in L_2[0, \infty) \cap L_\infty[0, \infty)$, which is a rather restrictive hypothesis. According to Theorem 1, then the following corollary can be stated:

**Corollary 1.** Consider the flexible spacecraft system in Eqs. (6b) and (8) satisfying Assumptions 1–3. With the application of a controller in Eq. (36),

$$u(t) = \frac{D^T}{\lambda} \left[ -k_2 \sigma - \left( ||S(\omega)DJ_w(\Omega + D^T\omega)|| + \frac{||\delta^T C \delta \omega|| + ||F|| \hat{\alpha}}{\sigma} \right) \right] + \sigma^T D E u(t) + \frac{1}{\mu_1} \hat{\alpha}$$

and the adaptive law in Eq. (27b), if the gain $k_3$ is selected such that $k_3 \geq ||d + Df||$ is satisfied, and with the other parametric constraints given in Eq. (27), then we can ensure that $x_1, x_2, \sigma, \hat{\alpha}, \xi \rightarrow 0$ as $t \rightarrow \infty$.

**Proof.** Consider the same Lyapunov function candidate $V_2$ as used earlier; the time derivative of $V_2$ along the system trajectories is given, after some algebraic operations, as

$$V_2 \leq -\frac{1}{2} k_1 x_1^2 x_1 - \xi^T Q \xi + \frac{1}{2} x_1^2 x_2$$

by introducing a small boundary $\epsilon > 0$.

In the above analysis, we assume that the disturbance $(d(t) + Df(t))$ is simply bounded, i.e., $(d(t) + Df(t)) \in L_2[0, \infty)$. We further suppose that $(d(t) + Df(t)) \in L_2[0, \infty) \cap L_\infty[0, \infty)$, which is a rather restrictive hypothesis. According to Theorem 1, then the following corollary can be stated:

**4. Simulation and comparison results**

The numerical application of the proposed control schemes to attitude control of a flexible spacecraft is presented using MATLAB/SIMULINK software, and the results are reported here. The spacecraft is characterized by a nominal main body inertia matrix [9]

$$J = \begin{bmatrix} 350 & 3 & 4 \\ 3 & 270 & 10 \\ 4 & 10 & 190 \end{bmatrix} \text{kg m}^2, \text{ and the coupling matrices }$$

$$\delta = \begin{bmatrix} 6.45637 & 1.27814 & 2.15629 \\ -1.25619 & 0.91756 & -1.67264 \\ 1.11687 & 2.48901 & -0.83674 \\ 1.23637 & -2.65814 & -1.12503 \end{bmatrix} \text{ kg}^{1/2} \text{ m/s}^2.$$
respectively; the first four elastic modes have been taken into account in the model used for simulating the spacecraft at \(\omega_n^1 = 0.7681, \omega_n^2 = 1.1038, \omega_n^3 = 1.8733, \omega_n^4 = 2.5496 \text{ rad/s} \) with damping \(\xi_1 = 0.0056, \xi_2 = 0.0086, \xi_3 = 0.013, \xi_4 = 0.025\), while for designing the controller only the first three mode have been involved. The inertia matrix \(J_w\) is selected as \(J_w = 10I_{4\times4} \text{ kg m}^2\), and the configuration matrix \(D\) of four reaction wheels is given as

\[
D = \begin{bmatrix}
1 & 1 & -1 & -1 \\
\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\
1 & 1 & 1 & 1 \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\
\sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} \\
\sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}.
\]

To test the controller performance with and without various
combinations of reaction wheel faults, the rest-to-rest maneuver is considered in the simulation, and the initial conditions have been set at \( q_0 = 0.173648, q_1 = -0.263201, q_2 = 0.789603 \) and \( q_3 = -0.526402 \), i.e., a rotation of 160° is to be considered in the attitude maneuvering. Note that, due to the change in reference, the controller regulates the quaternion to the equilibrium point \([1, 0, 0, 0]\), since this is the closest equilibrium point in terms of rotation path. In addition, the initial modal variables and the time derivatives \( \eta_i(0) \) and \( \dot{\eta}_i(0) (i = 1, 2, 3, 4) \) are supposed given by \( \eta_i(0) = \dot{\eta}_i(0) = 0 \), i.e., the flexible appendages are undeformed. In addition, in the simulation, the periodic disturbance torque

\[
d(t) = \begin{bmatrix} 0.3 \cos(0.01t) + 0.1 & 0.15 \sin(0.02t) + 0.3 \cos(0.025t) \\ 0.3 \sin(0.01t) + 0.1 \end{bmatrix}^T
\]

is also considered. In this study, three different sets of simulations are conducted to demonstrate the effectiveness of the proposed approach:

1. **Attitude control using the proposed control laws for a fault-free condition.**
2. **Attitude control with using proposed control law under fault scenario 1:**
   \[
   \begin{align*}
   f_2 &= 0.5, \quad 5 \leq t \leq 25 \text{ s} \\
   e_1 &= 0.2, \quad t \geq 3 \text{ s} \\
   e_3 &= 0.6, \quad t \geq 5 \text{ s} \\
   e_4 &= 0.4, \quad t \geq 10 \text{ s}.
   \end{align*}
   \]  
3. **Attitude control with using proposed control law under fault scenario 2:**
   \[
   \begin{align*}
   f_1 &= 0.3, \quad 2 \leq t \leq 10 \text{ s} \\
   f_3 &= 0.6, \quad 5 \leq t \leq 20 \text{ s} \\
   e_2 &= 0.2, \quad t \geq 10 \text{ s} \\
   e_3 &= 0, \quad t \geq 5 \text{ s} \\
   e_4 &= 0.8, \quad t \geq 2 \text{ s}.
   \end{align*}
   \]

In addition, for the purpose of comparison, the conventional proportional-derivative (PD) without reconfigurable process is also performed at the same simulation conditions. In the following simulations, the control and adaptation gains were selected by trial and error until a good performance was obtained for the above cases. The controller parameters of the different methods (proposed control law and PD) were determined so that all the settling times were almost the same in all the schemes for the fault-free condition.

### 4.1. Attitude control under a fault-free condition

In this case, first, to show the effect of the proposed adaptive backstepping sliding mode fault-tolerant controller in Eq. (27), the simulation was done under the given initial condition. The time histories of the quaternion of the spacecraft, angular velocity, modal displacement, required control torque and reaction wheel’s velocities are shown in Fig. 1 ((a)–(e), solid line). It is noted that an acceptable desirable orientation response is achieved, and the spacecraft reached the required angle with a settling time less than 30 s. Moreover, the elastic vibrations are significantly suppressed by considering them in the design, and the oscillations settle within 30 s. The last plot in Fig. 1 ((e), solid line) shows the vibration energy response, which is described by

\[
E = \eta^T \dot{\eta} + \eta^T K \eta.
\]

It can be observed that the energy has almost no oscillations after 30 s. This illustrates that the designed controller is capable of reducing the system vibration while maintaining the tracking capability of the spacecraft.

For the purpose of comparison, the system is also controlled by using the traditional PD control. The same simulation cases are repeated with this controller and the results of the simulation are shown in Fig. 2(a)–(e), solid line). For this case, it can be observed that the attitude rotational maneuver can be achieved, but severe oscillations are excited during maneuvering as demonstrated in the modal displacement and vibration energy responses as shown in Fig. 4((e), solid line). Even after maneuvering, the vibrations...
still exist. Despite the fact that there still exists some room for improvement with different design control parameter sets, there is not much improvement in the attitude and velocity responses. From the comparison between Figs. 1 and 2, the performance of the proposed design controller is better than that of the conventional PD control even if these designs will adapt the system parameters under external disturbances.

4.2. Attitude maneuver control under faulty conditions

Here two actuator fault scenarios are considered, as described by Eqs. (40) and (41), respectively. The first case is that a bias fault occurs at the second wheel for 20 s after 5 s after the simulation starts; the first, third and fourth wheels lose 80%, 40% and 60% of the control power after 3 s, 5 s, and 10 s after the simulation.

Fig. 3. Attitude maneuvering using the proposed sliding mode control method. Case 1: fault-free (solid line); Case 2: fault scenario 2 (dashed–dotted line).
starts, respectively. Figs. 1 and 2 ((a)–(e), dashed–dotted line) for this case show the results of the same motion commands used in the previous subsections for the proposed and PD control methods, respectively. It can be seen that high control precision and a good tracking process are still obtained for the proposed controller, and no significant amount of vibration occurred, whereas the PD controller shows the degradation of tracking performance after the actuator fault. In addition, the PD controller also shows tracking ability, to some degree, because of its own robustness.

The second case is that the third wheel loses complete control power after 5 s, a bias fault occurs at the first and third wheels for 8 s and 15 s after 2 s and 5 s respectively, and the second and fourth wheels loses 80% and 20% of the control power 10 s and 2 s after the simulation starts, respectively. This kind of combination is a harder fault scenario than the first case. As shown in Figs. 3 and 4 (a–3, dashed–dotted line), the proposed controller works effectively and achieves the control target, while for the PD controller, it can be observed that the system performance is significantly degraded after the actuator faults; moreover, severe vibrations result from the control law as compared with the proposed methods. What is more, the vibrations still exist during the tracking process, which deteriorates the performance of the attitude maneuvers.

Summarizing all the cases (normal case and fault cases), it is noted that the proposed controller design method can significant improve the normal performance over that of the PD method in both theory and simulations. Also, in the fault case, the proposed methods have better results than those of conventional cases. It can also be observed that as more and more faults are considered in the design, the proposed controllers can still guarantee the tracking performance. In addition, extensive simulations were also done using different control parameters, disturbance inputs and even a combination of reaction wheel faults. These results show that in the closed-loop system attitude control and vibration stabilization are accomplished in spite of these undesired effects in the system. Moreover, the flexibility in the choice of control parameters can be utilized to obtain desirable performance while meeting the constraints on the control magnitude and elastic deflection. These control approaches provides the theoretical basis for the practical application of the advanced control theory to a flexible spacecraft attitude control system.

5. Conclusions

A fault-tolerant adaptive backstepping sliding mode control scheme has been developed for flexible spacecraft attitude maneuvering using redundant (four) reaction wheels in the presence of parametric uncertainty disturbances and even unknown faults. The proposed control design methods do not require the system identification process to identify the faults as well as the process of fault detection and isolation. The control formulation is based upon Lyapunov’s direct stability theorem; the semi-globally input-to-state stability of the closed-loop system is ensured, and the robustness to disturbances, unknown faults and elastic vibrations is also guaranteed provided that appropriate robustness conditions on the controller gains are satisfied. The control designs have been evaluated using numerical simulation comparisons between the developed approach and other referenced schemes, and the expected performances have been shown to occur. In the simulations, several different types of reaction wheel failure scenarios were investigated. Based upon the results presented in the paper, it is concluded that the fault-tolerant adaptive backstepping sliding mode control scheme successfully handles failures if one reaction wheel fails completely, or if the efficiency of one or several reaction wheels decreases. While the simulation results presented in this paper merely illustrate formulations for a particular attitude maneuver, further testing would be required to reach any conclusions about the efficacy of the control and adaptation laws for tracking arbitrary maneuvers. In addition, this fault-tolerant control scheme places no restriction on the magnitude of the desired control, and a design explicitly considering the actuator limit is also being investigated. Future work is planned to study the digital implementation of such a control scheme on hardware platforms for attitude control experimentation.
(a) Time response of quaternion.
(b) Time response of angular velocity.
(c) Time response of reaction wheel’s velocities.
(d) Time response of control torque.

Fig. 4. Attitude maneuvering using the PD control method. Case 1: fault-free (solid line); Case 2: fault scenario 2 (dashed–dotted line).

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