Brief paper

Robust and adaptive variable structure output feedback control of uncertain systems with input nonlinearity

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Received 8 October 2005; received in revised form 21 December 2006; accepted 16 June 2007

Abstract

This brief proposes a robust control algorithm for stabilization of a three-axis stabilized flexible spacecraft in the presence of parametric uncertainty, external disturbances and control input nonlinearity/dead-zone. The designed controller based on adaptive variable structure output feedback control satisfies the global reaching condition of sliding mode and ensures that the system state globally converges to the sliding mode. A modified version of the proposed control law is also designed for adapting the unknown upper bounds of the lumped uncertainties and perturbations. The stability of the system under the modified control law is established. Numerical simulations show that the precise attitude pointing and vibration suppression can be accomplished using the derived robust or adaptive controller.

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Keywords: Dead-zone/nonlinearity; Flexible spacecraft; Variable structure control; Vibration suppression

1. Introduction

The fine attitude control of rigid and flexible structures during tracking maneuvers has been an important issue in spacecraft applications. The dynamics of large rotational and tracking maneuvers are time varying and nonlinear and contain bounded parameter uncertainties, and they are affected by various disturbances coming from environments. Therefore, the design of effective disturbance rejection control systems that are robust to parametric uncertainty poses a challenging task for spacecraft designers.

Variable structure control (VSC) is known to be an effective control technique applicable to a wide class of nonlinear systems subject to modeling uncertainty and external disturbances (Drakunov & Utkin, 1992). It has been applied to spacecraft attitude control problems in some previous studies (Bošković, Li, & Mehra, 2001; Chen & Lo, 1993; Hu & Ma, 2005; Iyer & Singh, 1991; Lo & Chen, 1995; Shen, Liu, & Hu, 2000; Zeng, Araujo, & Singh, 1999). However, in those works, VSC is limited to systems with full state feedback. In practical applications, full measurement of state might be neither possible nor feasible, for example, it is difficult to measure the variables describing the flexible motion, the modal position, and the velocity of flexible spacecrafts. Therefore, the direct output feedback design in VSC is worth investigating. Several authors have considered VSC with static output feedback (Heck & Ferri, 1989; Yallapragada, Heck, & Finney, 1996; Zak & Hui, 1993).

Another important problem encountered in practice is the control input nonlinearities, such as saturation and dead-zones, originating from actuators in system realization. The phenomenon often appears when the reaction wheel is used as the actuator for a spacecraft (Werta, 1978, Chapter 7). This kind of input nonlinearity is a source of performance degradation or even worse, instability of a system. Consequently, the problem of control design for systems with input nonlinearity has recently become an area of interests (Hsu, Wang, & Lin, 2004; Shyu, Liu, & Hsu, 2003). Hsu et al. (2004) has proposed VSC schemes for uncertain dynamic systems with dead-zone...
nonlinearity for solving regulation problems. However, the information of upper bound of uncertainties is required. In general, this bound is difficult to measure in practical applications. An adaptive approach may offer a simple and effective tool to solve this problem (Yoo & Chung, 1992).

The contribution of this brief is the design of a nonlinear controller to achieve the attitude maneuver of a three-axis stabilized flexible spacecraft while actively suppressing the vibrations of the flexible appendages under parametric uncertainty, external disturbances and control input nonlinearity/dead-zone. The proposed controller ensures the global reaching condition on the control torque and thus the control input nonlinearity/dead-zone in the input function is given in the form

\[
\phi(u(t)) = \begin{cases} 
  u_{\text{max}} & \text{if } u(t) > u_{\text{max}}, \\
  u(t) - u_{\text{max}} & \text{if } u(t) < u_{\text{min}}, \\
  u_{\text{min}} & \text{if } u(t) > u_{\text{min}}.
\end{cases}
\]

and such that 0 < \gamma u_{\text{max}}(u(t) - u_{\text{min}}) ≤ 1.

3. Robust adaptive variable structure output feedback control design

A linear switching surface is defined as

\[
S(t) = Gy = GCx(t),
\]

where \( S(t) = [s_1(t), s_2(t), \ldots, s_m(t)]^T \) is a vector of sliding surfaces and \( G = [g_1, \ldots, g_m]^T \in \mathbb{R}^{m \times p} \) is a specified constant matrix such that matrix GPU is nonsingular.

Let \( \hat{S}(t) = [\hat{s}_1(t), \hat{s}_2(t), \ldots, \hat{s}_m(t)]^T = B^T C^T G^T S(t) \), and then the following VSC law with nonlinearity/dead-zone is proposed:

\[
u_i = \begin{cases} 
\beta k \rho \hat{s}_i(t) \frac{\hat{s}_i(t)}{\| \hat{S}(t) \|} + u_0 & \text{if } \hat{s}_i(t) < 0, \\
0 & \text{if } \hat{s}_i(t) = 0, \\
-\beta k \rho \hat{s}_i(t) \frac{\hat{s}_i(t)}{\| \hat{S}(t) \|} - u_0 & \text{if } \hat{s}_i(t) > 0,
\end{cases}
\]

where \( k = \max(\| GCA \|, \| GCB \|) \), \( \beta > 1 \), \( \rho > 0 \), which will be defined later, and \( \hat{\gamma}(t) \) is generated as the solution of the linear differential equation

\[
\hat{\gamma}(t) = \beta k \rho \hat{s}_i^2(t) \| B^T C^T G^T S(t) \|
\]

with \( \hat{\gamma}(0) = \gamma_0 \). Note that \( \gamma_0 \) is the bounded positive initial value of \( \hat{\gamma}(t) \).

Lemma 1. For all the input nonlinearities \( \Phi(u) \) satisfying Assumption 3, the control law (7) satisfies that

\[
\hat{S}^T(t) \Phi(u(t)) \leq -\gamma k \rho \hat{s}_i^2 \| \hat{S}(t) \|^2,
\]

where 0 < \gamma < \gamma u_{\text{max}}(u_i(t) - u_{\text{min}}) ≤ 1.

Proof. From Eqs. (4) and (7), \( u_i > u_{i0} \) implies that \( \hat{s}_i(t) < 0 \) and thus

\[
(u_i - u_{i0}) \Phi(u_i) = \beta k \rho \hat{s}_i^2 \| \hat{S}(t) \|^2 \Phi(u_i) \leq -\gamma (u_i - u_{i0})^2 = \gamma u_{\text{max}}^2 \| \hat{S}(t) \|^2 \| \beta k \rho \hat{s}_i^2 \|^2,
\]

whereas for \( u_i < -u_{i0} \), \( \hat{s}_i(t) > 0 \) and thus

\[
(u_i + u_{i0}) \Phi(u_i) = \beta k \rho \hat{s}_i^2 \| \hat{S}(t) \|^2 \Phi(u_i) \leq \gamma (u_i + u_{i0})^2 = \gamma u_{\text{max}}^2 \| \hat{S}(t) \|^2 \| \beta k \rho \hat{s}_i^2 \|^2.
\]
Then
\[ \ddot{s}_i \Phi_i(u_i) \leq -\gamma \beta k \rho \ddot{s}_i(t) \frac{\ddot{s}_i^2(t)}{\|S(t)\|}. \] (11)

Therefore, the following inequality holds:
\[ \dot{S}^T(t) \Phi(u) = \sum_{i=1}^{m} \ddot{s}_i(t) \Phi_i(u_i) \leq \sum_{i=1}^{m} -\gamma \beta k \rho \ddot{s}_i(t) \frac{\ddot{s}_i^2(t)}{\|S(t)\|} \]
\[ = -\gamma \beta k \rho \ddot{s}_i(t) \|S(t)\|. \] (12)

Note that a nonsingular transformation of switching surface will not change the sliding mode dynamics. If a particular switching surface \( S_1 = G_1 y \) is chosen, it can be transformed to \( S = Gy \), where \( G = (G_1CB)^{-1}G_1 \). Therefore, without loss of generality, assume that \( GCB = I \) in the following.

**Theorem 1.** Consider the nonlinear system (1) subjected to Assumptions 1–3. If the input \( u(t) \) is given as that indicated by Eq. (7) with the adaptive control Eq. (8), then the system trajectories asymptotically converge to the sliding manifold \( S(y) = 0 \).

**Proof.** Consider a Lyapunov function candidate
\[ V(t) = \frac{1}{2} S^T S + \frac{1}{2} \ddot{s}_i^2(t), \] (13)
where \( \ddot{s}_i(t) = \ddot{s}_i^2(t) - \gamma \).

Then, the time derivative of \( V(t) \) has
\[ \dot{V}(t) = S^T GC [Ax + B \Phi(u) + B Hx + B d] \]
\[ + \ddot{s}_i(t) \ddot{s}_i(t) \leq k \|S\| (\|x\| + \|H\| + \|d\|) \]
\[ + S^T G CB \Phi(u) + \ddot{s}_i(t) \ddot{s}_i(t). \] (14)

Now define a constant \( \rho \) which always exists, satisfying
\[ \infty > \rho \geq (\|x\| + \|H\| + \|d\|) \] (15)
for all \( \infty > t \geq 0 \), and apply Lemma 1 and the assumption \( GCB = I \), we have
\[ \dot{V}(t) \leq k \rho \|S\| - \gamma \beta k \rho \ddot{s}_i(t) \|S(t)\| + \ddot{s}_i(t) \ddot{s}_i(t) \]
\[ = k \rho \|S\| - \gamma \beta k \rho \ddot{s}_i(t) + \ddot{s}_i(t) \beta k \rho \ddot{s}_i(t) \|S(t)\| \]
\[ = (1 - \beta) k \rho \|S\| \leq 0. \] (16)

If \( \ddot{s}_i(t) = (\beta - 1) k \rho \|S\| \) is defined, integrating the above equation from zero to \( t \) yields
\[ V(t) = V(0) + \int_{0}^{t} \ddot{s}_i(t) \, dt \geq \int_{0}^{t} \ddot{s}_i(t) \, dt. \] (17)

Taking the limit as \( t \to \infty \) on both sides of Eq. (17) gives
\[ \infty > V(t) \geq \lim_{t \to \infty} \int_{0}^{t} \ddot{s}_i(t) \, dt. \] (18)

Thus according to Barbalat lemma (Popov, 1973), we obtain
\[ \lim_{t \to \infty} \ddot{s}_i(t) = \lim_{t \to \infty} (\beta - 1) k \rho \|S\| = 0. \] (19)

Since \( \rho > 0 \) for all \( t > 0, \beta > 1, k > 0 \) and \( GCB \neq 0 \), Eq. (19) implies \( S(y) \to 0 \) as \( t \to \infty \). Hence the proof is achieved completely.

**4. Modified adaptive variable structure output feedback control design**

In Section 3, we have presented a robust adaptive variable structure output feedback control for uncertain systems with saturation nonlinearity and dead-zone where the bounds of the uncertainties and disturbances are known. In this section, we shall address a modified adaptive variable structure output feedback when these bounds are unknown.

We now propose the following adaptive variable structure output feedback control:
\[ u_i = \begin{cases} -\beta k \ddot{s}_i(t) \ddot{s}_i(t) + u_0 & \text{if } \ddot{s}_i(t) < 0, \\ 0 & \text{if } \ddot{s}_i(t) = 0, \\ -\beta k \ddot{s}_i(t) \ddot{s}_i(t) - u_0 & \text{if } \ddot{s}_i(t) > 0, \end{cases} \] (20)
with \( \ddot{s}_i(t) \) given as
\[ \ddot{s}_i(t) = \beta k \ddot{s}_i^3(t) \|B^T C^T G^T S(t)\|. \] (21)

Note that \( \ddot{s}_i(t) \) is the estimate of \( \rho \), and \( \ddot{s}_i(t) \) can be obtained from the following dynamics:
\[ \ddot{s}_i(t) = \ddot{s}_i(t) - \ddot{s}_i(t), \] (22)
where \( \ddot{s}_i(t) > 0 \) and \( \ddot{s}_i(t) > 0 \) is the positive and bounded initial value of \( \ddot{s}_i(t) \).

Let \( \ddot{s}_i(t) = \ddot{s}_i(t) - \ddot{s}_i(t) \) denote the adaptation error. Because \( \ddot{s}_i(t) \) is assumed to be constant, then the following expression keeps valid:
\[ \ddot{s}_i(t) = \ddot{s}_i(t). \] (23)

Based on the above parameter adaptive controller, we obtain the following theorem.

**Theorem 2.** Consider the nonlinear system as described in Eq. (1). If the modified law is designed as Eq. (20) with Eqs. (21) and (22), and Assumptions 1–3 hold, then the trajectory of the nonlinear system converges to the sliding manifold \( S(y) = 0 \).

**Proof.** Consider the Lyapunov function candidate as \( \dot{V}(t) = \dot{V}(t) + (\ddot{s}_i(t)/2) \ddot{s}_i(t) \ddot{s}_i(t), \ddot{s}_i(t) > 0 \). Using Eq. (20) and the same manipulations as Eqs. (14)–(16), the time derivative of \( \dot{V}(t) \) becomes
\[ \dot{V}(t) \leq k \ddot{s}_i(t) \ddot{s}_i(t) \|S(t)\| + \ddot{s}_i(t) \ddot{s}_i(t) + \ddot{s}_i(t) \ddot{s}_i(t) \leq (1 - \beta) k \ddot{s}_i(t) \ddot{s}_i(t) \|S(t)\|. \] (24)

For the moment, using the result of Theorem 1, we can conclude that \( S(y) \to 0 \) as \( t \to \infty \). Hence the proof is achieved completely.

**Remarks.** In this section, the modified adaptive version of the proposed adaptive variable structure output feedback control law has been also designed for adapting the unknown upper bounds of the lumped uncertainties and perturbations so that the limitation of knowing the bound of the uncertainties and perturbations in advance is released.
5. Application to a flexible spacecraft

5.1. Mathematical model

The dynamics of spacecraft with flexible appendages can be obtained from the Lagrangian approach (Gennaro, 1998). With the assumption of a small Euler angle rotations, the dynamic model can be approximated as

\[
\begin{bmatrix}
(J_T - J_R) & \delta^T \\
\delta & I
\end{bmatrix}
\begin{bmatrix}
\dot{\Delta s} \\
\Delta s
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 \\
0 & D
\end{bmatrix}
\begin{bmatrix}
\dot{\phi} \\
\dot{\psi}
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 \\
0 & K
\end{bmatrix}
\begin{bmatrix}
\phi \\
\theta
\end{bmatrix}
= \begin{bmatrix}
I & 0
\end{bmatrix}(\Sigma(\cdot) - u_3),
\]

where \(\phi, \theta\) and \(\psi\) are roll, pitch and yaw attitude angles, and \(\Sigma(\cdot)\) is a function of the moments of inertia of the flexible spacecraft, the coupling matrix between attitude and vibrations modes, etc.

Introducing a new variable

\[z = [\phi \, \theta \, \psi \, \eta]^T\]

and setting

\[\tilde{M} = \begin{bmatrix}
(J_T - J_R) & \delta^T \\
\delta & I
\end{bmatrix}, \tilde{D} = \begin{bmatrix}
0 & 0 \\
0 & D
\end{bmatrix}, \tilde{K} = \begin{bmatrix}
0 & 0 \\
0 & K
\end{bmatrix}, \tilde{B} = [I \, 0]^T,
\]

the following equation can be obtained:

\[\tilde{M}\ddot{z} + \tilde{D}\dot{z} + \tilde{K}z = \tilde{B}[\Sigma(\cdot) - u_3].\]

Define the state variable \(x_s = [\phi \, \theta \, \eta \, \dot{\phi} \, \dot{\psi} \, \dot{\eta}]^T \in R^{6+3}\), the state-space spacecraft model describing the dynamics behavior of the flexible spacecraft can be described by

\[
\dot{x}_s(t) = (A_s + \Delta A_s)x_s(t) + B_s\phi_s(u_s) + f_s(t),
\]

\[y_s = C_s x_s,
\]

where

\[
A_s = \begin{bmatrix}
0 & -\tilde{M}^{-1}\tilde{K} & -\tilde{M}^{-1}\tilde{D}
\end{bmatrix}, \quad B_s = \begin{bmatrix}
0 \\
\tilde{M}^{-1}\tilde{B}
\end{bmatrix},
\]

\[f(t) = B_s\Sigma(\cdot), \quad C_s = \begin{bmatrix}
I_{3 \times 3} & 0_{3 \times \bar{n}} & 0_{3 \times \bar{n}} & 0_{3 \times \bar{n}} & 0_{3 \times \bar{n}} & 0_{3 \times \bar{n}}
\end{bmatrix}.
\]

5.2. Simulation results

In this section, application of the proposed control schemes to the attitude control of a flexible spacecraft is presented using MATLAB/SIMULINK software. The nominal moments of inertia parameters, coupling matrices, and the natural frequencies are taken from Gennaro (1998).

Suppose the initial values of the three attitude angles are \(\phi(0) = 6^\circ, \theta(0) = 4^\circ\), and \(\psi(0) = -4^\circ\). The control objective is to drive each of the three attitude angles to zero from their initial value within \(t_f = 150\) s, and each of the vibration mode coordinates satisfies \(\eta_i(0) = \eta_i(t_f) = 0\), \(i = 1, 2, 3, 4\). Suppose that the matched uncertainty and disturbance are of the forms: \(\Delta A_s = B_e(1 + \sin x_s)\) and \(f_s = B[\sin 5t]\), respectively.

In the numerical simulation, for comparison, three cases are conducted: (1) attitude maneuver control using the proposed robust adaptive variable structure output feedback controller (7); (2) attitude maneuver control using the traditional variable structure output feedback controller given in Yallapragada et al. (1996) as follows:

\[u = Ny - \beta(GCB)^{-1}\frac{S}{\|S\| + \varepsilon}\]

and (3) attitude maneuver control using the modified adaptive variable structure output feedback controllers (20). The attitude response and vibration suppression are analyzed to study the performances of the controllers.

5.2.1. Robust adaptive variable structure output feedback control (RAVSOFC)

Fig. 1 shows the results of the proposed robust adaptive variable structure output feedback controller in the presence of uncertainty and disturbance. From Fig. 1(a), it can be seen that each of the three attitude angle responses (solid line) almost approaches zero around the time of 100 s, and the steady error of each angle is less than 0.002\(^\circ\). Fig. 1(b) shows the responses (solid line) of angular velocity of the flexible spacecraft and the steady errors are no more than 0.002\(^\circ\)/s. Responses of the vibrations modes are illustrated in Fig. 1(d). The maximum induced vibration coordinate of the first mode is effectively suppressed. It is noted that all the vibration modal coordinates and vibration rates approach zero at time 10 s.

5.2.2. Traditional variable structure output feedback control (TVSOFC)

For the purpose of comparison, the system employed by TVSOFC law in Eq. (32) proposed by Yallapragada et al. (1996) is also considered in this section. The same simulation case is repeated with the TVSOFC replacing the proposed RAVSOFC for a fair comparison and the results of simulation were also shown in Fig. 1 (dotted line). For this case, a significant amount of the oscillations of the attitude angle and rate occurred during the maneuvering as demonstrated in Figs. 1(a) and (b) (dotted line) with steady errors more than 0.05\(^\circ\) and 0.05\(^\circ\)/s, respectively. Moreover, the oscillations do not settle within 300 s. The disturbance rejection is markedly worse than that achieved.
Fig. 1. Attitude maneuver control using proposed RA VSOFC and TVSOFC. (a) Responses of attitude angle. (b) Responses of attitude angle rate. (c) Coordinate responses of first–fourth vibration modes.
by the proposed approach. In addition, from the dotted line as shown in Fig. 1(c), TVSOFC also introduces much more vibrations to the first four modes than proposed RA VSOFC and the maximum induced vibration coordinate of the first mode is nearly close to 0.5 kg/\sqrt{m}.

### 5.2.3. Adaptive variable structure output feedback control

For the modified adaptive control case in Eq. (20) with adaptive laws (21) and (22), the same tests are also repeated with the same control parameters and the simulation results are shown in Fig. 2. In this case, the attitude angles and rates response of

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**Fig. 2.** Attitude maneuver control using modified AVSOFC. (a) Responses of attitude angle. (b) Responses of attitude angle rate. (c) Coordinate responses of first–fourth vibration modes.
each axis are shown in Figs. 2(a) and (b), respectively. As expected, the responses of both the angle displacement and the angular velocity almost also converge to zero in 100 s. The maximum induced vibration coordinate of the first mode is effectively suppressed to below 0.1 kgm/2 m as shown in Fig. 2(c). It is seen that the adaptive control law, as expected, is essentially capable of suppressing the vibrations while maintaining the attitude maneuvering capability of flexible spacecraft even after the information of upper bound of the perturbations and uncertainties is not required in advance.

6. Conclusions

In this paper, the problem of vibration suppression of three-axis stabilized flexible spacecraft during attitude maneuver in the presence of bounded model uncertainty, external disturbances and control input nonlinearity/dead-zone has been investigated. A robust adaptive variable structure output feedback controller to stabilize uncertain dynamics system with explicitly considering nonlinearity/dead-zone control input has been proposed. It is shown that the designed controller guarantees the global reaching condition of the sliding mode for the uncertain system. In addition, we have also proposed a modified adaptive version of this algorithm, which removes the constraint of known parametric uncertainty bound and disturbance bound. Simulations have verified the effectiveness of the proposed designs.

Acknowledgments

This present work was supported by National Natural Science Foundation of China (Project Number: 60674101) and Research Fund for the Doctoral Program of Higher Education of China (Project Number: 20050213010). The authors would like to thank the reviewers and the Editor for many suggestions that helped improve the paper.

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